Evaluation of the accuracy of the SAGEOID 2010 model in the Cape Peninsula

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Abstract

Survey practitioners are challenged with the dilemma of whether or not to utilize the SAGEOID 2010 model in topographical surveys because they are unsure of the height accuracy attainable from its application. This paper presents an evaluation of the accuracy of the SAGEOID 2010 model in two case study areas in Cape Town. In order to achieve this, the objective was to investigate and compare a relative and absolute verification in terms of height of SAGEOID 2010 model and to clarify the achievable accuracy thereof.

The main purpose of this article is to discuss the accuracy of the SAGEOID 2010 model using GNSS positioning techniques. The SAGEOID 2010 model was used in two study areas in Cape Town within a Virtual Reference Station (VRS) network cluster. The model was verified in both the absolute and relative sense. The results suggest that in the absolute sense, the accuracy improves with shorter baseline lengths from the Continuously Operating Reference Stations (CORS). The results showed a mean of 0.004m and RMS of 0.021m in Rondebosch compared to a mean of 0.023m and RMS of 0.029m in Parow. The two types of verifications confirmed the published accuracy of the SAGEOID 2010. There has been very little published work on the SAGEOID 2010 and this study is intended to contribute towards the geodetic body of research.

1. Introduction

1.1 Background

The surface of a geoid is important to engineering and geosciences as a physically defined datum used to determine orthometric heights. The geoid has been described differently by various authors. Torge (2001:3) cited the mathematician Gauss and stated that:

"What we call the surface of the earth in the geometrical sense is nothing more than that surface which intersects everywhere the direction of gravity at right angles, and part of which coincides with the surface of the oceans."

Fotopoulos (2003) offered the classical definition of the geoid stating that it is:

"an equipotential surface of the Earth's gravity field that coincides with the mean sea level".

Notably, although this definition is widely accepted, some studies have found that the mean sea level departs from the equipotential surface by up to two meters due to various oceanographic phenomena such as variable temperature, salinity, instantaneous sea surface topography etc. (Vaníček and Krakiwsky, 1986).

1.2 The SAGEOID

The SAGEOID 2010 is a hybrid geoid model created from combining a geometric model (GPS/levelling data) with a gravimetric model. It covers the whole of South Africa on a 2.5' grid in a region between latitudes -35° and -22° and longitudes 16° and 33° respectively (Figure 1).

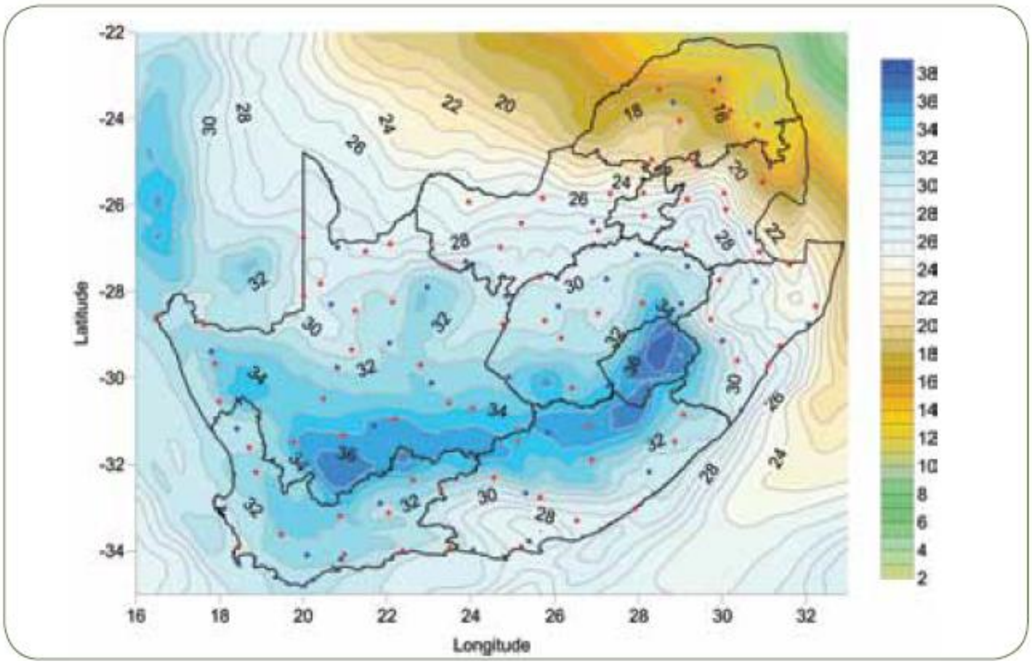

Figure 1. The South African Geoid Model 2010 (metres) (Chandler & Merry, 2011)

The accuracy of the geoid model can be expressed in the absolute and relative sense. The SAGEOID 2010 has an accuracy of 7cm in the absolute sense (Chandler and Merry, 2010). This implies that the heights derived from the SAGEOID 2010 could be up 7cm too high or too low. However, in the relative sense where a height is determined from a control point in relatively close proximity, it is expected that the results after the SAGOEID 2010 is applied at the base station will be better than those observed in the absolute sense.

In their study on the SAGEOID 2010 Wonnacott & Merry (2011) assessed the potential for using a precise geoid model as an alternative vertical datum. It was found that based on 1500 trigonometrical beacons with GNSS derived ellipsoidal heights, a mean difference of 0.22m with a standard deviation of 0.47m was obtained when compared to the orthometric heights determined primarily via trigonometrical heighting.

Further, the minimum and maximum height differences ranged from -2.86m to +4.59m. The comparison was intended to highlight the low accuracy of trigonometrical heighting. The figure below illustrates a graphical representation of the differences.

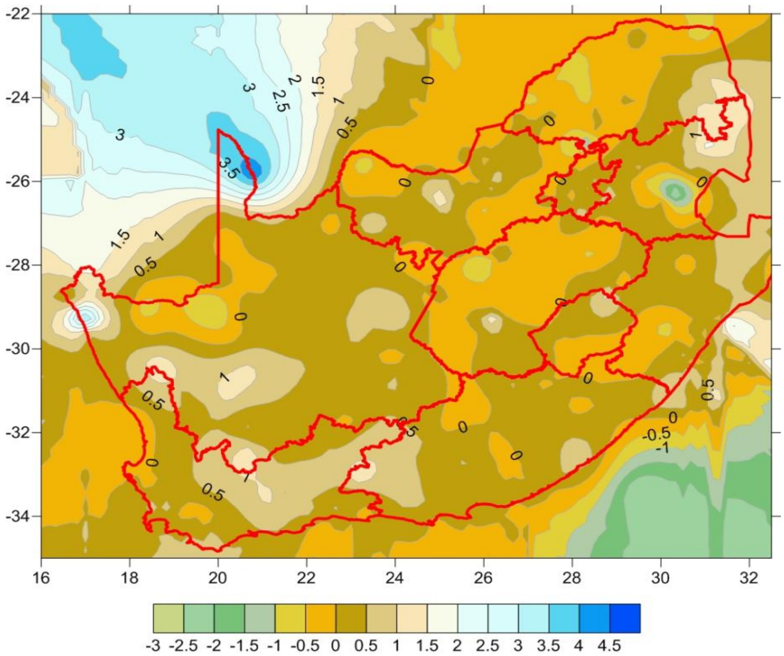

Figure 2. Differences between published orthometric heights of trigonometrical beacons and orthometric heights using the SAGEOID2010 (metres) (Wonnacott & Merry, 2011)

In relation to this, surveyors in private practice are challenged with the dilemma of whether or not to utilize the SAGEOID 2010 because of uncertainty around the accuracy of heights attainable from its application. Most surveyors rely on a geometric geoid model (orthometric heights) by using sufficient known points (benchmarks) instead of using a single point calibration with the SAGEOID 2010 applied. Reason being, it can present a high risk of inaccuracy when using a single point calibration with no SAGEOID 2010 applied. This is especially the case when there are insufficient benchmarks available such as when practitioners conduct topographical surveys for road construction over long distances in remote areas. In this instance it is essential that heights determined are accurately measured with the use of SAGEOID 2010 model.

Over the last few decades, there has been an increased interest in the gravimetric determination of the geoid. This includes the growing demand for a refined geoid model for users of GNSS receivers, who must transform GPS-derived ellipsoidal heights to orthometric heights in order to make them compatible with the existing orthometric heights on the local vertical datum (Featherstone, 2001). Moreover, since satellite based positioning techniques such as Global Navigation Satellite Systems (GNSS) are now commonly being used in a wide range of geodetic and surveying applications, it has become more important to accomplish a transformation between ellipsoidal heights and orthometric heights. When using GPS, the determined positions are in relation to a geocentric WGS84 (World Geodetic System 1984) reference ellipsoid and the ellipsoid surface is assumed as the datum of heights which are derived from GPS measurements (Erol and Çelik, 2004). The following section briefly explains the difference between the ellipsoidal height and orthometric height surfaces.

1.3 The relationship between ellipsoidal height and geoidal undulation

The equation below shows the relationship between ellipsoidal heights obtained from GPS measurements and orthometric heights established from spirit levelling data with gravimetric corrections is as given in the equation (Heiskanen and Moritz, 1967).

$$h - H - N = 0 \quad [1]$$

The terms in the equation above represent the following:

h = ellipsoidal height, H = orthometric height, N = geoid undulation obtained from a regional gravimetric geoid model or a global geopotential model. The ellipsoidal height *h* is measured orthogonal to the ellipsoid. The orthometric height *H* is measured orthogonal to the Geoid.

The following figure illustrates the relationship between ellipsoidal height and the geoid undulation:

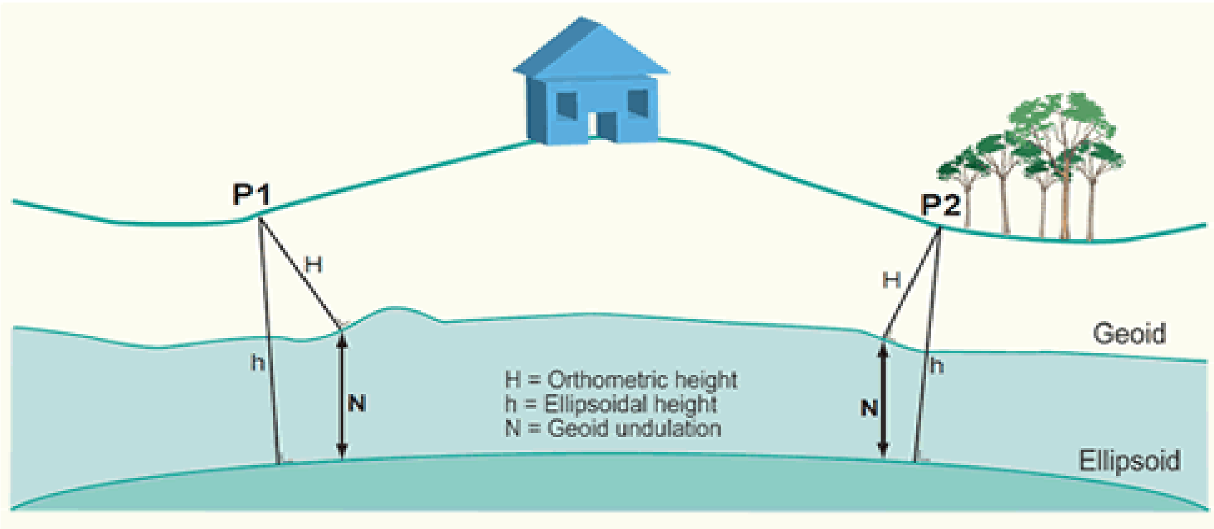

Figure 3.Relationship between the ellipsoid and the geoid (http://kartoweb.itc.nl/geometrics/Reference%20surfaces/refsurf.html, viewed March 2014)

For the relative case, where height differences are considered, we simply have:

$$\Delta h - \Delta H - \Delta N = 0 \quad [2]$$

1.4 Factors that affect Geoid Modelling

There are a number of common error sources that affect the quality of all three ellipsoidal coordinates. For instance, satellite geometry or configuration especially when only observed in one hemisphere can cause errors (Rothacher, 2002).

Erol and Çelik (2004) state that the following important factors affect the accuracy of the GPS/Levelling geoid model:

a) The distribution and number of reference stations (GPS/Levelling stations). These points must be distributed homogeneously to the coverage area of the model and have to be chosen to figure out the changes of geoid surface.
b) The accuracy of the GPS derived ellipsoidal heights (h) and the heights derived from levelling measurements (H).
c) The characteristics of the geoid surface in the area.
d) The method used while modelling the geoid (research prescribes various models for modelling the geoid surface of different areas).

The characteristic appeal of the geometrical relationship between these three height types is based on the principle that if any two heights are given, the third height can be derived through simple manipulation of equation [1] or similarly equation [2] in the relative instance (Fotopoulous, 2003). However, the practical application of the equations above is more complex due to various factors that cause inconsistencies when combining the different heights (Rummel and Teunissen, 1989; Kearsley *et al.,* 1993; Schwarz *et al.,* 1987). The major aspect of the various discrepancies is usually ascribed to the systematic errors and datum inconsistencies. When dealing with these effects it entails the incorporation of a parametric model (commonly known as corrector surface) in the collective adjustment of the heights (Fotopoulous, 2003). Several studies have been performed using this approach with many different types of parametric models from a simple bias, a bias and a tilt, higher order polynomials with different base functions (Shretha *et al.*, 1993), finite element models (Jäger, 1999), Fourier series (Haagmans *et al.,* 1998) and collocation-based approaches (Forsberg & Madsen, 1990 in Fotopoulous, 2003). Hence, it is not practical to have a universal model that is applicable in all cases since it is evident from these studies that the appropriate type of corrector surface model will differ depending on the height network data (Fotopoulous, 2003).

A study was conducted in the northern part of Algeria involving the assessment of the precision of orthometric heights determined by combining GPS data with the local geoid model. The study revealed that the standard deviation of the absolute differences between the GPS and levelling derived geoid heights and the gravimetric geoid was approximately 0.68m. Systematic biases were also found between the GPS and levelling derived geoid heights and the gravimetric geoid with a mean value of -1.481m. These systematic biases are a result of datum discrepancies common among the different height types (Daho, 2010). These include:

a) a different reference surface and long wavelength systematic errors in N
b) poorly modelled GPS errors (i.e. tropospheric effects)
c) distortions in the orthometric height datum due to an over-constrained adjustment of the levelling network
d) effect of various geodynamic effects,
e) assumptions and theoretical approximations made in processing observed data
f) improper or non-existent terrain/density modelling in the geoid modelling
g) orthometric heights and negligence of the sea surface topography at the tide gauges

The major part of these discrepancies is usually attributed to long wavelength errors which are introduced with the use of a global model in gravimetric geoid computation that is not necessarily optimal for the area of interest (Fotopoulous, 2003). In this instance a corrector surface model can be used to reduce the discrepancies (Lin, 2014).

1.5 Validation of the Geoid

According to Abdallah and Fairhead (2011) the gravimetric geoid model can be verified in the absolute and relative sense using the spirit levelled orthometric heights (data commonly used to validate gravimetric geoid). In the absolute verification, the accuracy and precision of the gravimetric geoid with respect to the geocentric ellipsoid can be estimated using GPS networks that have been connected to an international reference frame as well as spirit-levelled orthometric heights that have been connected to a local vertical datum. This approach can also be used to constrain the zero-degree spherical harmonic term that is deficient in any gravimetric geoid model computed using Stokes's integral (Featherstone, 2001).

In relation to the aforementioned paragraph, absolute verification measurements can be obtained using Virtual Reference Stations (VRS). One requirement for setting up a virtual reference station is that it must be located less than 60km from other reference stations with precisely known coordinates. In some parts of South Africa the reference stations may exceed the prescribed threshold distance of 60km if situated in areas with minimal multipath. Equipment may also be used to mitigate the effects of multipath. Incidentally, Real Time Kinematic (RTK) surveys are more prone to the effects of multipath since solutions are resolved from only one base station. In VRS surveys, since the coordinates are precisely known there is no need to resolve for the baseline or wavelengths. In addition, with long occupation times the ionosphere and troposphere can be interpolated and accurately modelled between reference stations. It is for this reason that a VRS survey can be used within a VRS network cluster and slightly beyond. The integer ambiguity can be resolved once the baseline distances, real time orbits and clock as well as the modelled troposphere and ionosphere are known. With all this information known a virtual base station can be simulated close to where the rover/receiver is located. This facilitates rapid baseline resolution and initialization times. All reference stations are used within a network cluster when doing a VRS survey and shorter baselines are allocated higher weights when calculating the solution for the virtual base station (Parker, pers comm, October 2013).

In contrast, during relative verification the accuracy and precision of the gravimetrically computed geoid gradients is estimated from the GPS-derived ellipsoidal height differences and the spirit levelled orthometric height differences (equation [2]). Verification in the relative sense simply assesses the accuracy and precision of the computed geoid gradients. Since most GPS surveys are not normally conducted in an absolute sense, this type of assessment would be more farmiliar to the majority of the survey practitioners using the SAGEOID 2010. To conduct a validation in a relative sense, the user must constrain GPS measurements to the local vertical datum. This is because the geoid height used at the GPS base station is cancelled during differencing (Featherstone, 2001). The primary advantage of this approach is that errors common to either end of the control baseline cancel on differencing (Kearsley, 1988). For instance, errors relating to the vertical datum are cancelled to a large extent especially over short distances. However, in the instance of older vertical datums containing localised geodetic levelling errors, this may not be so easily satisfied (Featherstone, 2001).

2. Research Design and Methodology

2.1 Research design

The objective of this research was to evaluate the accuracy of the SA GEOID 2010 model using relative and absolute verification. The methods used for verification are based on prescriptions in the preceding text (Fotopoulous, 2003; Abdallah & Fairhead, 2011). A combination of comparative research and experimental research techniques was used. The research was carried out in two areas in Cape Town each having a network of published orthometric points. After gathering enough GPS field data in the two test areas the data was compared to the corresponding orthometric heights.

2.1.1 Study areas

The selection of areas chosen for investigation was based on the following criteria:

a) The areas had to have a height difference between them that was more than the prescribed accuracy of the SAGEOID 2010. This was assessed from the geoid contour maps which show the level of undulation in an area of interest.
b) Availability of points with published orthometric heights such as Town Survey Marks (TSMs).
c) The TSMs had to be orientated perpendicular to the direction of change in geoidal undulation (Desai, 2011).

Based on these criteria it was decided that the two test areas would be located in two different suburbs namely Parow and Rondebosch. The height difference between the two suburbs is approximately 10cm

2.2 Research methodology

2.2.1 Research instruments / equipment

Dual frequency GNSS receivers were used for the field work in this study. The relative and absolute surveys were done concurrently and each point was occupied over a period of 30 seconds. The SAGEOID 2010 was applied and 20 measurements were taken on each controller/logger.

2.2.2 Relative Accuracy Measurements

As stated earlier, the two study areas were chosen because they are densely populated with benchmarks which could be used to test the geoidal accuracy relative to the 10cm change in height of the SAGEOID 2010. The Parow Valley site measured approximately 1.5km diagonally from North-West to South-East. In Rondebosch, the length of the site measured approximately 1.2km from East to West and 0.7km from North to South.

For the relative verification, measured data was acquired using a Real-Time Kinematic (RTK) survey and at least two receivers were used. For this research study, the receiver that was used as a base was set up over a town survey mark (TSM) with a published orthometric height and positioned with the known coordinates of the set up point. The TSMs used as bases were the ones closest to the centre of each of the study areas. The orthometric height was constrained, and the geoid was applied at the base and the ellipsoidal height was then determined at the base. This is used with the vector (ellipsoidal height difference) to determine the ellipsoidal height at the rover/receiver. The orthometric height was then determined at the rover by applying the geoid on the computed ellipsoidal height at the rover. This process was repeated in order to measure the heights of the surrounding TSMs with known orthometric heights.

When the TSMs were far from the base or when the radio signal was out of range a repeater was introduced to boost the signal in order facilitate the survey. However, after a few measurements in the Rondebosch study area, initialization problems were encountered when using the Trimble R6 receiver/rover. In order to overcome this problem, a decision was made to use the Trimble R4 receiver for the relative measurements. The survey took longer than anticipated because both absolute and relative measurements were done on one rover for the rest of the points in the Rondebosch study area. In order to achieve this, survey styles were continuously being switched between RTK and VRS on each TSM.

2.2.3 Absolute Accuracy Measurements

The SAGEOID2010 reference frame is ITRF2008, epoch 2012, which is the same reference frame that three dimensional coordinates (and ellipsoidal heights) of the current TrigNet network are transmitted from. TrigNet is a network of permanent CORS distributed throughout South Africa at approximately 100km to 300km spacing. Absolute measurements were obtained using a VRS survey with a Trimble R4 receiver. When using VRS survey style, multiple CORS are used to obtain a fixed solution or position within a VRS network environment. In Figure 4, the red lines indicate the baseline lengths from the nearest CORS. Data was compared within each study area in order to assess how accurate the SAGEOID 2010 is in that specific study area.

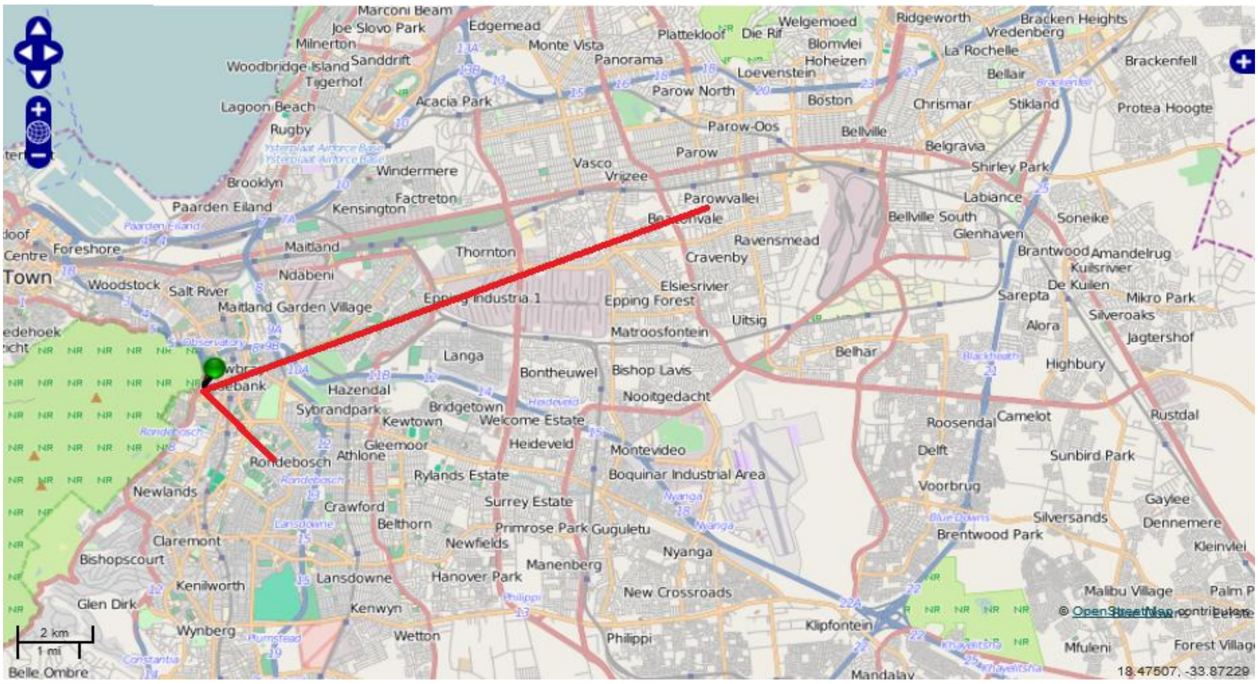

Figure 4. Sensor map showing the lengths of the baselines from the nearest CORS at Mowbray.

3. Results

The following figure shows a comparison of the residuals based on relative and absolute GPS observations at the Town Survey Marks. They are a graphical representation of the results obtained.

Table 1. Comparison between relative and absolute accuracy in the Parow study area.

	Relative (23 OBS)	**Absolute (23 OBS)**
Mean with TSM 358O5	0.016m	-0.015m
RMS with TSM 358O5	0.046m	0.045m
STD with TSM 358O5	0.044m	0.043m
	Relative (22 OBS)	**Absolute (22 OBS)**
Mean without TSM 358O5	0.007m	-0.023m
RMS without TSM 358O5	0.012m	0.029m
STD without TSM 358O5	0.010m	0.017m

Table 1 shows the mean, root mean square (RMS) and standard deviations of the residuals from the relative and absolute verification carried out in Parow. The residuals at one point, TSM 358O5, were found to be inconsistent with the other TSMs i.e. 0.167m and 0.211m in the absolute and relative verification respectively. Nonetheless the table shows the results including and excluding observations to TSM 35805.

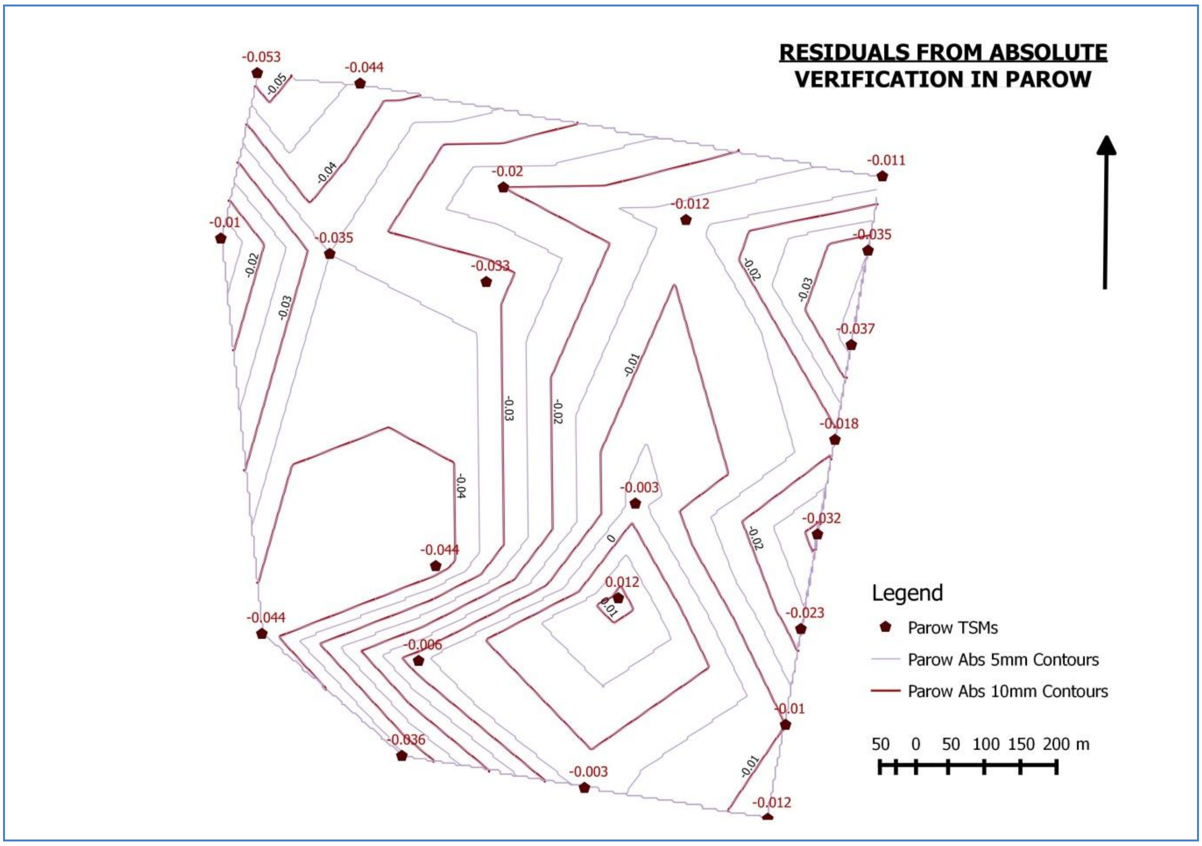

Figure 5. Contours showing residuals from absolute verification in the Parow study area.

In the absolute verifications most of the residuals in the South of the site were negative which shows a consistent gradient (Figure 5). There was one value of +0.012m in the South which does did not follow the trend and caused a spike in the contours. This may be attributed to random noise. A similar trend is found in the relative verification where almost all the residuals are positive with negative residuals only occurring at the site boundary. Based on the surrounding values and since the residuals are on the boundary this could either suggest a change in gradient or random noise.

The residuals from the relative verification are smaller than those from the absolute verification (Table 1 and Figure 6). From Figure 5 the Town Survey Marks in the South West of the test area are the closest to the CORS at Mowbray. Notably, the distance between the individual TSMs and the CORS did not have an impact on the residuals (Figure 5). The residuals from the absolute verification are consistent with most of them between -0.020 and -0.050. The contours would have shown a similar result but they are skewed by the +0.012m residual from the TSM 355O5 located in the South of the test area (Figure 5). This could be treated as random noise at that observation since no other observations are consistent with that finding. The distance from the base station did not have an effect on the residuals from the relative verification (Figure 6). It is also worth mentioning that TSM 349O5 which was used as the base station for the relative verification survey had a residual of -0.005m in the absolute verification.

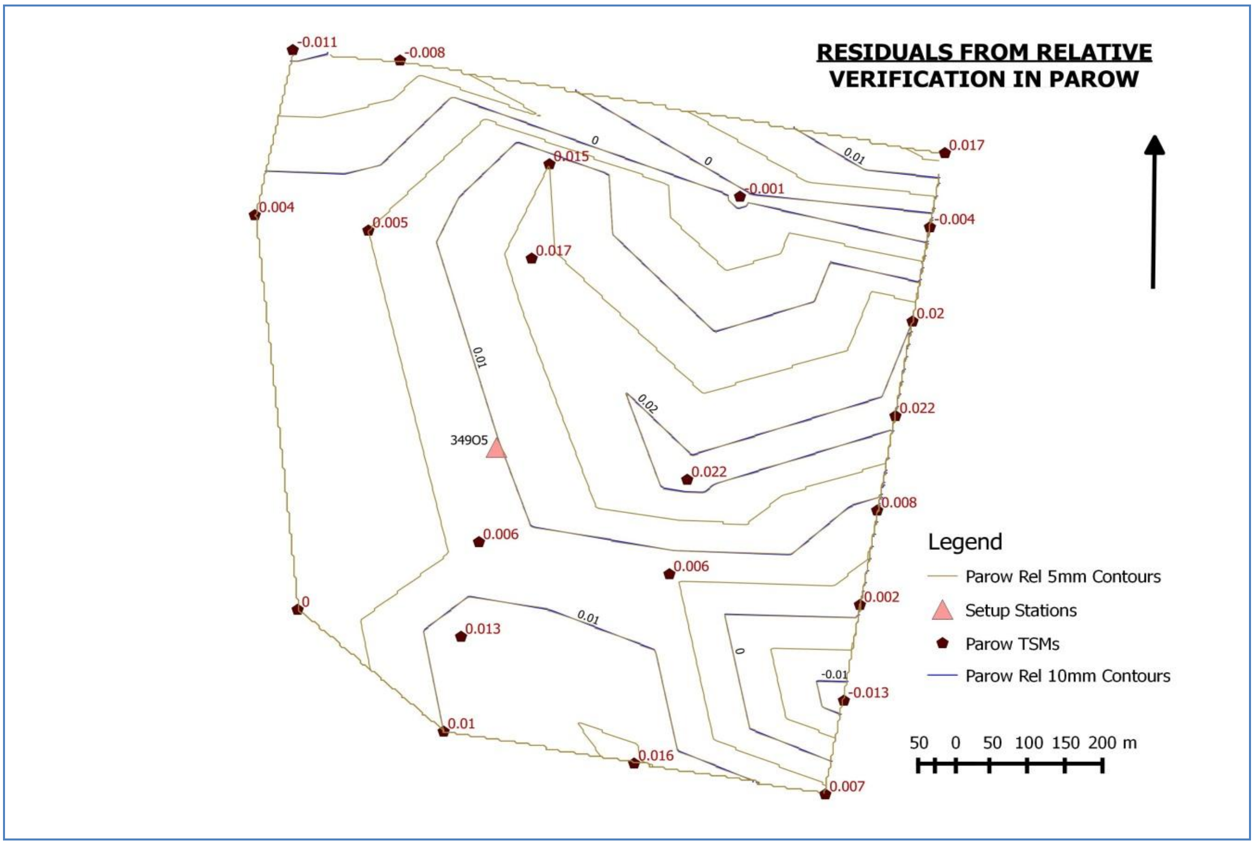

Figure 6. Contours showing residuals from relative verification in the Parow study area.

Two TSMs namely 2O13 and 5O14 could not be measured due to interference and density of trees in some parts of the Rondebosch area.

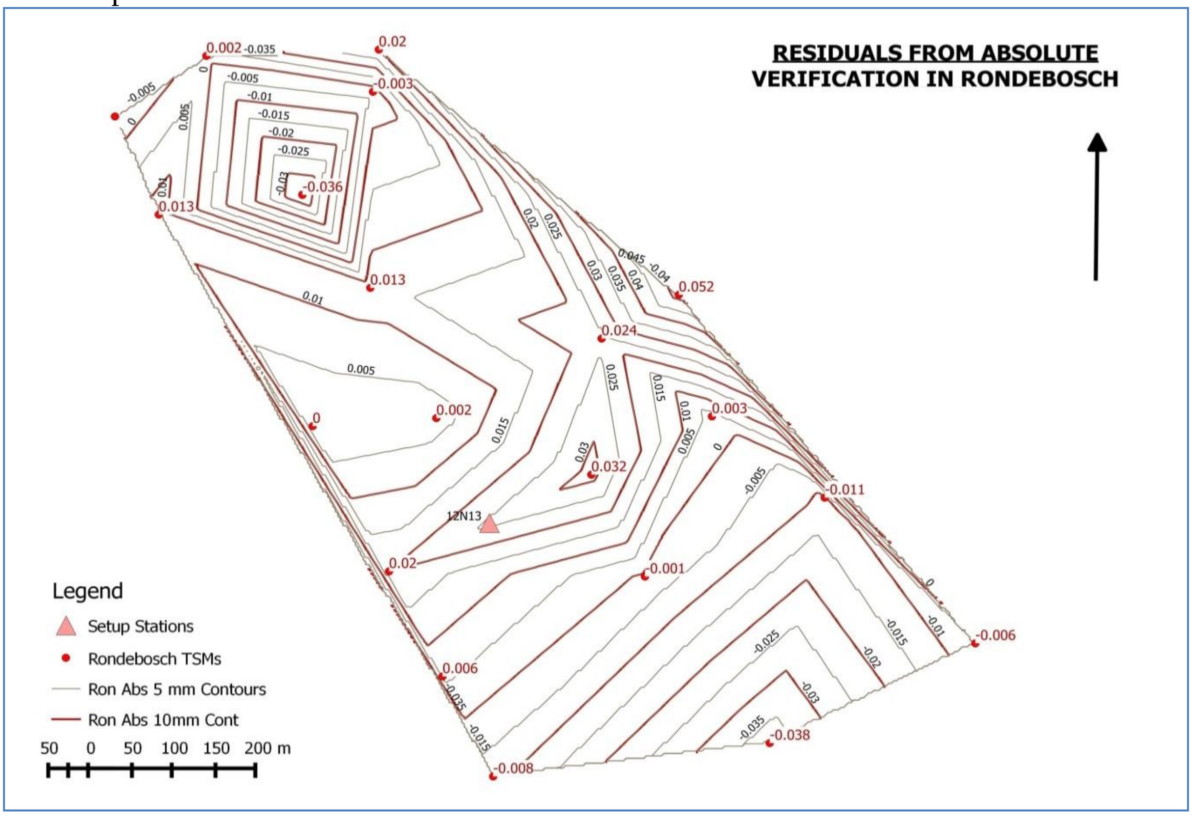

Figure 7. Contours showing residuals from absolute verification in the Rondebosch study area.

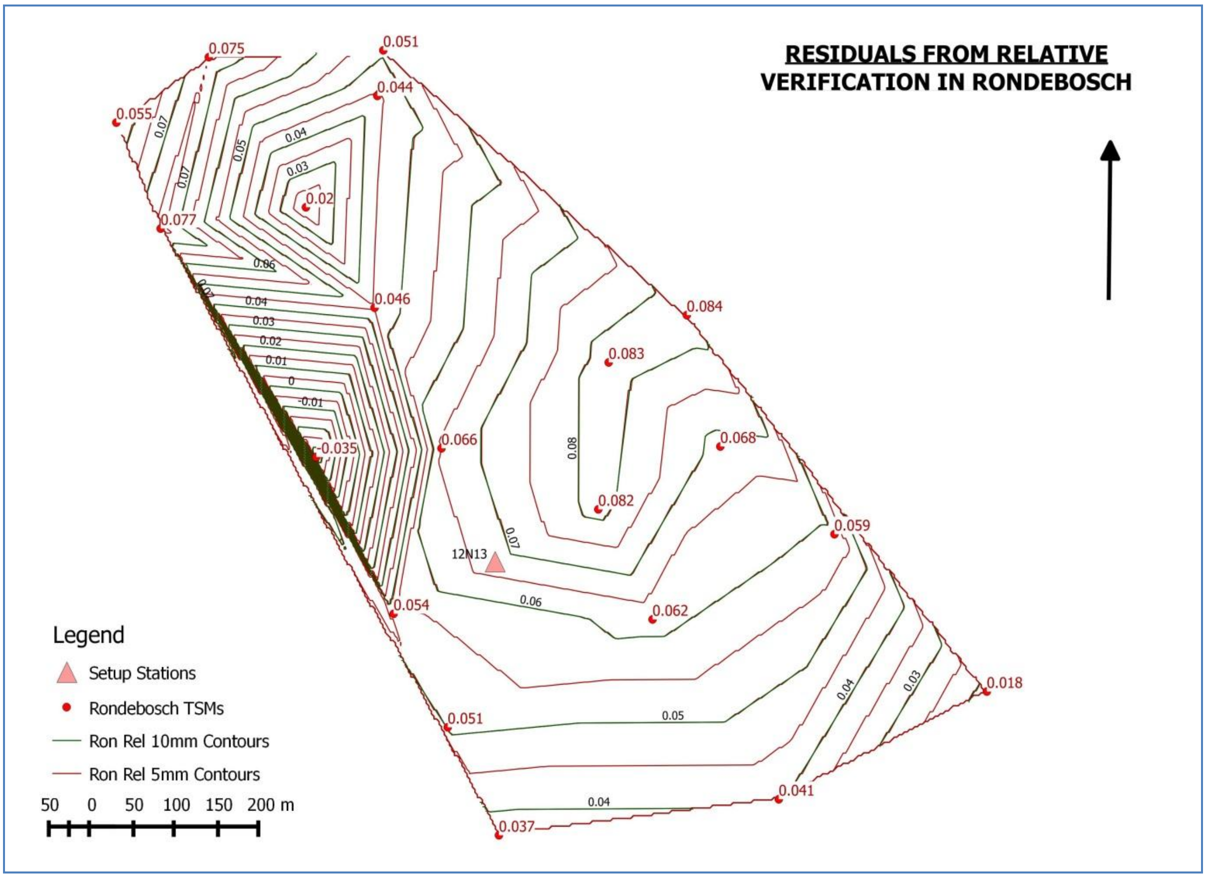

Figure 8. Contours showing residuals from relative verification in the Rondebosch study area.

Table 2. Comparison between relative and absolute accuracy in the Rondebosch study area.

	Relative (21 OBS)	**Absolute (21 OBS)**
Mean with TSM 4N13	-0.238m	0.012m
RMS with TSM 4N13	1.319m	0.043m
STD with TSM 4N13	1.330m	0.043m
	Relative (20 OBS)	**Absolute (20 OBS)**
Mean without TSM 4N13	0.052m	0.004m
RMS without TSM 4N13	0.059m	0.021m
STD without TSM 4N13	0.028m	0.021m

TSM 4N13 was inconsistent with the rest of the observations with a large residual of 0.176m (Absolute) and -6.040m (Relative). Table 2 also shows the mean, RMS and the standard deviation with and without TSM 4N13. In the absolute verifications most of the residuals in the South of the site were negative and those in the North were positive which could indicate a change in gradient (Figure 7). There was one value of -0.036 which does not follow the trend and may also be caused by random noise. A similar trend is found in the relative verification where almost all the residuals are positive. The steep slope in the contours is also caused by one negative residual at the Western boundary of the site which could also suggest random noise rather than a change in gradient.

It was also found that the residuals from the relative verification (Figure 7) were greater than those of the absolute verification measurements (Figure 8). The anomaly may be explained by the methodology used for the survey. Ideally, the TSM with smallest residuals from the absolute measurements should have been used as the base for the relative surveys. However, since the surveys were done simultaneously, it is possible that TSM 12N13 which was used as a base in Rondebosch was not optimal. The TSM may have been disturbed. It is also worth noting that observations to the station produced a residual of -0.029m in the absolute verification.

Based on the orientation of the site (Figure 5) the TSMs in the North West of the test area are the closest to the CORS at Mowbray. In comparison to the Parow site, it was also found that the distance between the individual TSMs and the CORS did not have an impact on the residuals (Figure 7). However the residuals for the absolute verification at Rondebosch were less than those from Parow i.e. a mean of 0.004m and RMS of 0.021m in Rondebosch compared to a mean of 0.023m and RMS of 0.029m in Parow. Therefore, the length of the baseline may have an effect on the residuals in the absolute verification.

Considering both test areas, the highest residuals were found in the relative verification at Rondebosch. The maximum residual was 0.084m. However this result should be investigated further considering that most of the residuals in the relative verification of Rondebosch were between 0.030m and 0.080m and the base station had a residual of -0.029m from the absolute verification. Nonetheless a residual of 0.084m is still an acceptable result considering that authors have stated that there are also slight differences between the levelling datum and the geoid (Daho, 2010; Chandler & Merry, 2011).

Lastly, the respective GNSS instrument specifications for the equipment used for this research are 8mm + 0.5ppm horizontally and 15mm + 0.5ppm vertically. Based on this the horizontal and vertical error estimate to the nearest base station would be 13.6mm and 20.6mm respectively for Parow. The corresponding horizontal and vertical error estimates for Rondebosch are 9.4mm and 16.4mm respectively. These values could have affected the results from the absolute verification.

4. Conclusions and Recommendations

The study generally found consistent trends in the residuals of the both sites. This would suggest a good estimation of the gradient of the SAGEOID 2010. The spikes in the contours sometimes indicated a change in gradient or possible random noise. Nonetheless the residuals were still within the published accuracy of the SAGEOID 2010.

The fact that the absolute verification in Rondebosch had smaller discrepancies than the relative verification could be because the chosen base station was not ideal. The absolute misclosure at this

TSM was -0.029m. It is recommended that this site is revisited and another TSM is used as a base.

Absolute verification was carried out in this research using a VRS survey style and it was found that the results may be dependent on the length of baselines. This can be seen in the values in Table 1 and Table 2 where the mean and RMS values are smaller for Rondebosch than Parow. Nonetheless, the results of the absolute verification were still less than the published accuracy of the SAGEOID 2010.

Due to time constraints only two study areas were chosen for research. Testing the SA GEOID 2010 in the greater part of South Africa with more study areas would give a better understanding of the geoid model. Also, there was only had a 10cm geoidal height difference between the study areas and the study could have been improved by assessing areas with steeper gradients.

It is recommended that two bases or base setups are used during relative verification surveys. This would serve as a check to ensure that the published orthometric heights are still valid. Alternatively, the potential base stations may also be verified with spirit-levelling or direct EDM measurement techniques.

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